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Combined radiation and natural convection in a two-dimensional participating square medium

ZHIQIANG TAN and JOHN R. HOWELL

Department of Mechanical Engineering, The University of Texas at Austin, Austin, TX 78712, U.S.A.

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Abstract—Combined radiation and natural convection in a two-dimensional emitting, absorbing and isotropically scattering square medium is studied numerically. The exact integral formulation for radiant transport and the momentum and energy balance equations are discretized by the product-integral method and finite difference method, respectively. The resulting algebraic equations are solved by a non-linear SOR technique, with the Rayleigh number varying from 10^3, 10^4 to 10^5, and the radiation–conduction parameter ranging from 0 to ∞. The influences of radiation–conduction parameter, Rayleigh number and other parameters on flow and temperature distributions and heat transfer are discussed.

INTRODUCTION

COMBINED radiation and natural convection in enclosures is important in many areas such as fire spreading and building insulation systems. Various pure natural convection problems have been investigated by numerous experimental and numerical researchers. A review paper can be found in Ostrach [1] and a comparative study of various numerical solutions of the natural convection in a two-dimensional square cavity was given by de Vahl Davis and Jones [2]. Benchmark solutions of the latter problem are also available [3].

Several works are found for the problem of combined radiation and natural convection in participating media. Larson [4] studied fire-spreading processes in buildings using a numerical approach. Chang *et al.* [5] investigated combined radiation and natural convection in two-dimensional enclosures with partitions. Desreyaud and Lauriat [6] computed natural convection and radiation in rectangular enclosures using a one-dimensional $P-1$ radiation analysis. Webb and Viskanta [7] measured the natural convection induced by irradiation and compared experimental results with the results of an analysis based on a spectral one-dimensional radiation model. A review paper on this subject was given by Yang [8]. Recently, Yucel *et al.* [9] studied coupled natural convection and radiation in a rectangular participating medium using finite differences and the two-dimensional discrete ordinates ($S4$ and $S8$) method. They included the effect of enclosure tilt and internal uniform sources on the results, but treated only the case of black boundaries. Natural convection and radiation in a participating medium between concentric cylinders were studied in ref. [10]. In all these works, radiation is found to play an important and sometimes major role in heat transfer and fluid flow processes.

In this work, combined radiation and natural convection in emitting, absorbing, and isotropically scattering square cavities is studied. The exact formulation for radiation is applied and discretized by the product integration method (PIM) [11]. The non-linear successive-over-relaxation (NSOR) iterating scheme for combined radiation and convection–conduction heat transfer problems [12, 13] is used. Comparisons are made with existing work for limiting cases. Influences of various parameters on heat transfer, temperature, and fluid flow are discussed.

FORMULATION

Consider a square emitting, absorbing, and isotropically scattering medium bounded by two horizontal insulating walls, and two vertical isothermal walls at different temperatures, T_h, and T_c ($T_h > T_c$), respectively (Fig. 1). For simplicity, all the physical properties in the system are assumed constant, except for the density which varies in the Boussinesq sense. The participating medium is assumed gray, and the emissivities of the walls are assumed to be the same.

Scaling the temperature (T) by T_c, length (x or y, where the $-y$-axis is in the gravity direction) by L, and velocity (u or v) by α/L, the governing Navier–Stokes equations in dimensionless stream function–vorticity form are

$$\nabla^2\psi = -\omega \quad (1a)$$

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y} = Pr\,\nabla^2\omega+Pr\,Ra\frac{\partial T}{\partial x} \quad (1b)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \nabla^2 T+Rc\,\nabla\cdot\mathbf{q}_r \quad (1c)$$

where Pr is the Prandtl number, Ra the Rayleigh number, Rc the radiation–conduction parameter defined by

$$Rc = L\sigma T_c^3/\lambda$$

and $\nabla\cdot\mathbf{q}_r$ is the radiation energy flux vector, which

NOMENCLATURE

e_g	radiative emissive power of medium
e_s	radiative emissive power of enclosure
L	side length of the medium
$\mathbf{n}$	inward unit normal vector at $\mathbf{r}$
Pr	Prandtl number
$\mathbf{q}_c$	convection heat flux on S
$\mathbf{q}_r$	radiation flux vector
q_s	radiation heat flux on S
Q, Q_{comb}	scaled total heat flux
$\mathbf{r}$	position vector, (x, y)
Ra	Rayleigh number
Rc	radiation–conduction parameter, $L\sigma T_c^3/\lambda$
S	boundary of Ω
S_n	Bickley's function of nth order
T	temperature
T_h, T_c	temperatures on hot and cold walls
u, v	x- and y-components of velocity.

Greek symbols

α	thermal diffusivity
ε	emissivity
λ	thermal conductivity
σ	Stefan–Boltzmann constant
τ	optical depth
ψ	stream function
ω	vorticity; albedo for scattering
Ω	medium.

relates equation (1) with the radiation transport equations of integral form [12]

$$4e_g(\mathbf{r}) - \frac{1}{1-\omega}\nabla\cdot\mathbf{q}_r(\mathbf{r}) = \int_\Omega \left[e_g(\mathbf{r}') - \frac{\omega}{4(1-\omega)}\nabla\cdot\mathbf{q}_r(\mathbf{r}')\right]\frac{S_1(\tau|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}A(\mathbf{r}') + \int_S \left[e_s(\mathbf{r}') - \frac{1-\varepsilon}{\varepsilon}q_s(\mathbf{r}')\right]\frac{S_2(\tau|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}\cos(\mathbf{r}-\mathbf{r}',\mathbf{n}')\,\mathrm{d}l(\mathbf{r}'),\quad \mathbf{r}\in\Omega \tag{2a}$$

$$e_s(\mathbf{r}) - \frac{1}{\varepsilon}q_s(\mathbf{r}) = \int_\Omega \left[e_g(\mathbf{r}') - \frac{\omega}{4(1-\omega)}\nabla\cdot\mathbf{q}_r(\mathbf{r}')\right]\frac{S_2(\tau|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}\cos(\mathbf{r}'-\mathbf{r},\mathbf{n})\,\mathrm{d}A(\mathbf{r}') + \int_S \left[e_s(\mathbf{r}') - \frac{1-\varepsilon}{\varepsilon}q_s(\mathbf{r}')\right]\frac{S_3(\tau|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} \times\cos(\mathbf{r}'-\mathbf{r},\mathbf{n})\cos(\mathbf{r}-\mathbf{r}',\mathbf{n}')\,\mathrm{d}l(\mathbf{r}'),\quad \mathbf{r}\in S \tag{2b}$$

where $\Delta x_j \equiv x_j - x_j'$ $(j = 1, 2)$, and S_i are Bickley's functions defined by

$$S_i(t) = \frac{2}{\pi}\int_0^{\pi/2} \mathrm{e}^{-t/\cos\theta}\cos^{i-1}\theta\,\mathrm{d}\theta$$

$\Omega \equiv (0, 1)\times(0, 1)$ is the region of the medium, and S the boundary of Ω.

The boundary conditions are

$$T = T_h/T_c,\quad x = 0;\quad 0 \leqslant y \leqslant 1 \tag{3a}$$

$$T = 1,\quad x = 1;\quad 0 \leqslant y \leqslant 1 \tag{3b}$$

$$q_c + Rc\,q_s = 0,\quad y = 0, 1;\quad 0 < x_2 < 1 \tag{3c}$$

$$\psi = 0,\quad \text{on all boundaries} \tag{3d}$$

where $q_c \equiv -\partial T/\partial n$ is the conductive heat flux on the wall, and q_s the radiative heat flux on the wall.

In this work, we take $T_h/T_c = 2$ and $Pr = 0.71$.

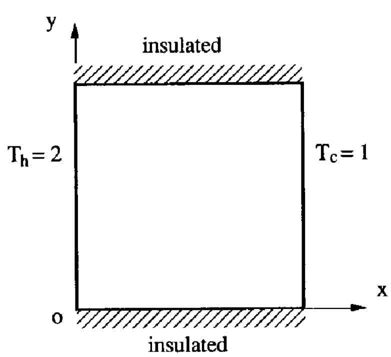


FIG. 1. Geometry of the problem.

NUMERICAL METHOD

The medium domain is divided equally into 25×25 square elements, and the four boundaries are divided into 4×25 equal segments. The nodes are located at the centers of each element or segment. An additional four nodes lie at the four corners, so there are 27×27 nodes in total. This mesh number is not large enough for high accuracy modeling for natural convection at high Rayleigh numbers, but it is large enough for accurate radiation modeling. Increasing the number of nodes increased the time of solution for the flow equations.

Equations (1) are discretized by conventional centered, second-order finite difference schemes. The derivative $-\partial T/\partial n$ in equation (3c) is approximated as a three-point, second-order accurate finite difference. The radiation transport equations (equations (2)) are discretized using the product integration method with piecewise-constant interpolants [11].

To solve the global non-linear algebraic equations, the non-linear SOR iterating scheme by Chen [14] and improved by Tan [12] is applied. This technique has been successfully used to solve several coupled radiation and convection (or conduction) problems [10, 13].

To provide convenient scaling of the results, the total heat flux on the wall, $q_c + Rc\ q_s$, is divided by $1 + Rc$

$$Q \equiv Q_{\text{comb}} \equiv \frac{q_c + Rc\, q_s}{1 + Rc}.$$

Similarly, we define the scaled radiative and convective fluxes as

$$Q_r = \frac{Rc\, q_s}{1+Rc} \quad \text{and} \quad Q_c \equiv \frac{q_c}{1+Rc}.$$

The convergence criterion used is

$$\frac{\max|\psi^n - \psi^{n-1}|}{\max|\psi^n|} + \max|T^n - T^{n-1}| < 10^{-5}$$

where the superscript n denotes the nth iteration.

All computations were performed on a CDC 170/750 system at the University of Texas at Austin.

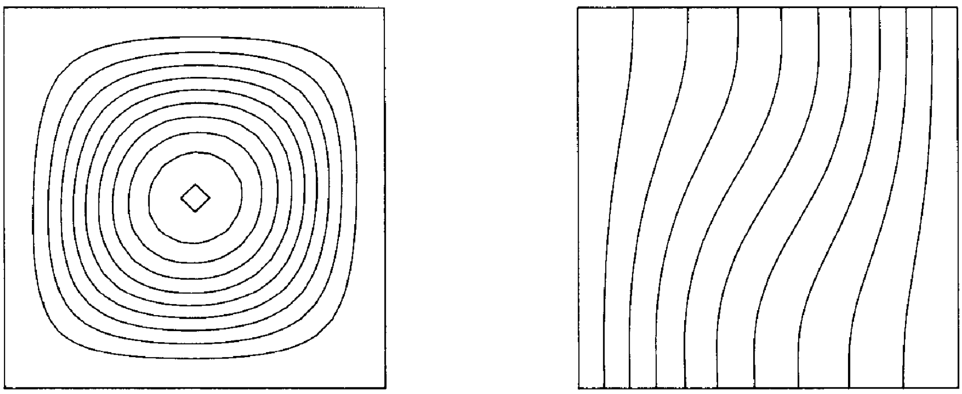
RESULTS AND DISCUSSIONS

When T_h/T_c and Pr are fixed, there are five independent parameters in this problem: Ra, Rc, ε, ω, and τ. It is difficult to perform complete investigations on the effects of all combinations of these parameters. Therefore, only trends are discussed.

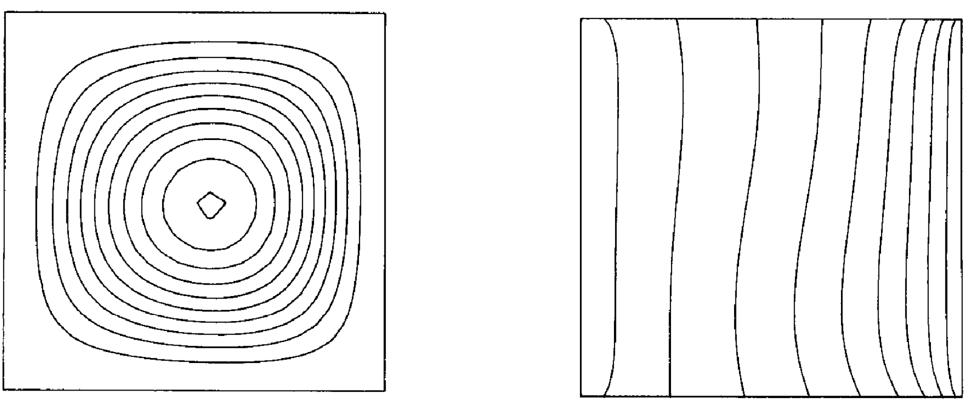
Results were obtained at $Ra = 10^3$, 10^4 and 10^5. The case $Ra = 10^6$ was also tried, but the accuracy was poor due to the coarse mesh. Solutions for the limiting case $Ra = 0$ (i.e. combined radiation and conduction) were given in another paper [13].

Influence of radiation on flow and temperature distributions

Figures 2–4 show the influence of the radiation–conduction parameter, Rc, on flow and temperature distributions, when $\omega = 0$, $\varepsilon = 1$ and $\tau = 1$. It is seen that when radiation is present, the flow and temperature distributions are no longer anti-symmetric as in the pure natural convection case, due to the nonlinearity of the energy equation (equation (1c)). At $Rc = 1$, the temperature gradients are found larger near the cold wall than for the pure convection case. This indicates that the bulk temperature is increased by radiation. In fact, due to the fourth power law of

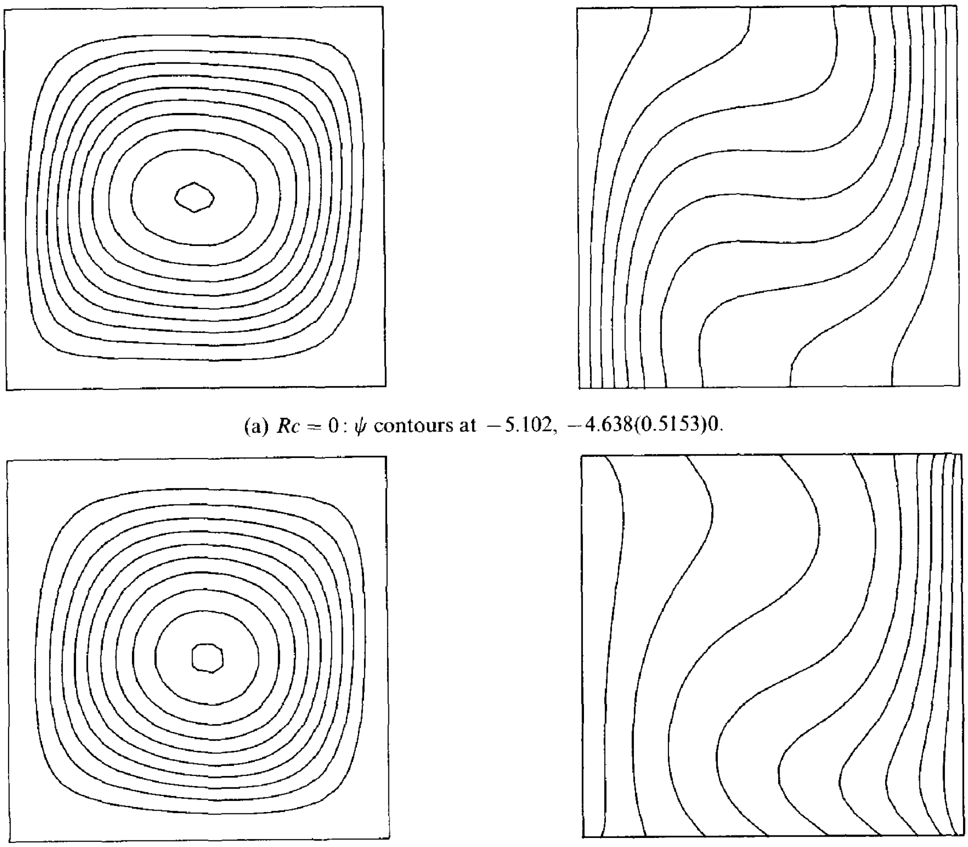


(a) $Rc = 0$: ψ contours at -1.172, $-1.065(0.1182)0$.



(b) $Rc = 1$: ψ contours at -0.945, $-0.859(0.0955)0$.

FIG. 2. Streamline and temperature contours when $Ra = 10^3$.



(a) $Rc = 0$: ψ contours at -5.102, $-4.638(0.5153)0$.

(b) $Rc = 1$: ψ contours at -6.916, $-6.287(0.6989)0$.

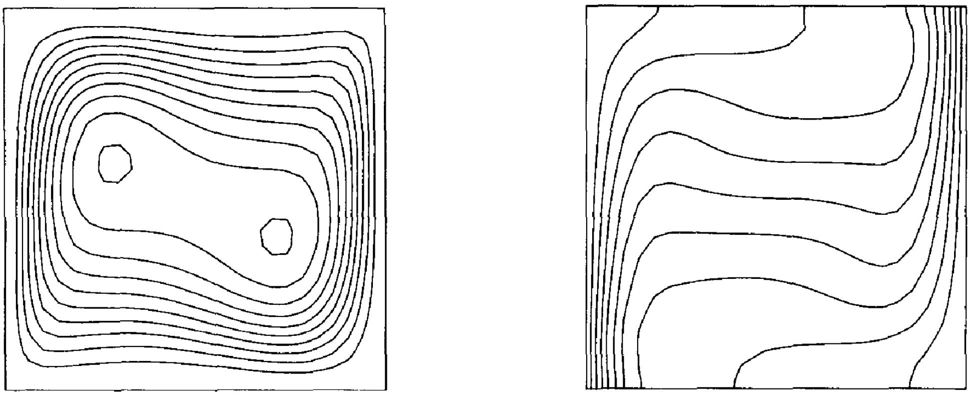
FIG. 3. Streamline and temperature contours when $Ra = 10^4$.

radiation, the temperature at the center of the enclosure, $T(0.5, 0.5)$, which is 1.5 at the pure convection limit should be increased to $[(1^4+2^4)/2]^{1/4} = 1.7075$ at the pure radiation limit as Rc increases (Fig. 5). When radiation is present, the temperature contours near both the hot and cold walls are found to be more parallel to the y-axis, implying that the heat flux on the vertical walls is more uniform.

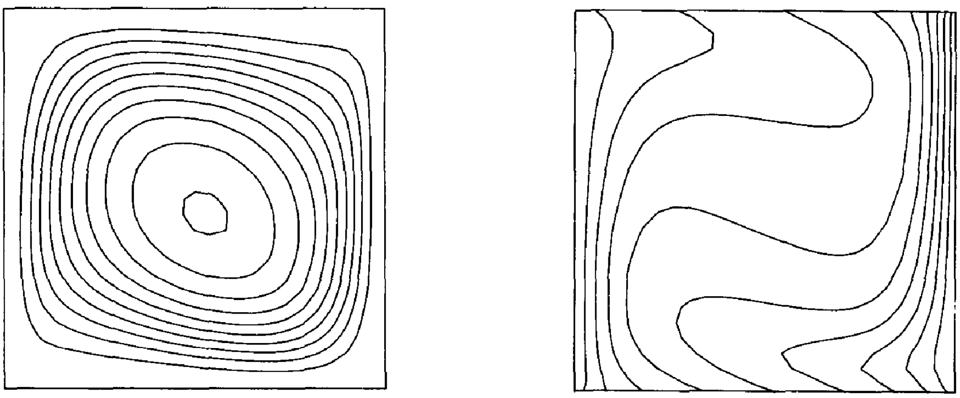
In the present problem the flow pattern is directly coupled with the temperature distribution in the cavity. As Rc increases, the flow field has less influence on the temperature distribution. In the limiting case $Rc = \infty$, the temperature field is independent of the flow field (and therefore Ra and Pr), while the flow pattern varies with Ra. (Mathematically, for those who are interested in dealing with only this limiting radiation-induced convection problem, the energy equation may be solved first, and the temperature distributions obtained are then used to solve the flow equations. This is in contrast with solving a forced convection problem with temperature-independent properties, where the flow equations are solved first.) Therefore, the influence of the radiation–conduction parameter Rc on the flow field is significant. For example, as Rc increases from 0 (pure convection) to 1, the cores are pushed from the center to lower right when $Ra = 10^3$ and 10^4; greater change is found at $Ra = 10^5$, where the original two separate cores are combined into one.

When the Rayleigh number increases or decreases, the influence of Rc on the maximum $|\psi|$, and hence the total circulating mass rate, is not monotonic. When $Ra = 10^3$, the maximum $|\psi|$ ($= 1.184$) is greater than the maximum $|\psi|$ ($= 0.5625$) for pure radiation; and when Rc increases, the maximum $|\psi|$ decreases monotonically (not shown here). However, when $Ra = 10^4$, a different pattern of Rc vs maximum $|\psi|$ line was observed. The maximum $|\psi|$ at $Rc = \infty$ is greater than at $Rc = 0$; and as Rc increases, maximum $|\psi|$ increases first until $Rc = 1.7$, then it decreases (Fig. 6). When $Ra = 10^5$, on the other hand, the maximum $|\psi|$ simply increases as Rc increases. These results are due to the complex interactions between the fluid mechanics and the combined radiative and conductive transfer. It is not clear that any general conclusions can be drawn which will apply, for example, at different optical thicknesses. A similar phenomenon was found for combined radiation and natural convection in an annulus between concentric cylinders [10].

Figure 7 depicts the y-component velocity on the horizontal mid-plane when $Ra = 10^4$. It is seen that the



(a) $Rc = 0$: ψ contours at -9.752, $-8.865(0.9850)0$.



(b) $Rc = 1$: ψ contours at -18.676, $-16.979(1.8865)0$.

FIG. 4. Streamline and temperature contours when $Ra = 10^5$.

line is smoothened when the system is more radiative. The line with the largest magnitude is found at $Rc = 1$.

Influence of radiation on heat transfer

Figures 8(a) and (b) show the influence of the radiation–conduction parameter on heat flux distributions on the hot wall ($x = 0$) and the cold wall ($x = 1$), respectively, when $Ra = 10^4$. Since in the pure radiation case the heat flux on each wall is symmetric, the wall flux distributions are more symmetric as the radiation tends to be more important. This was also predicted in connection with the results of Fig. 5.

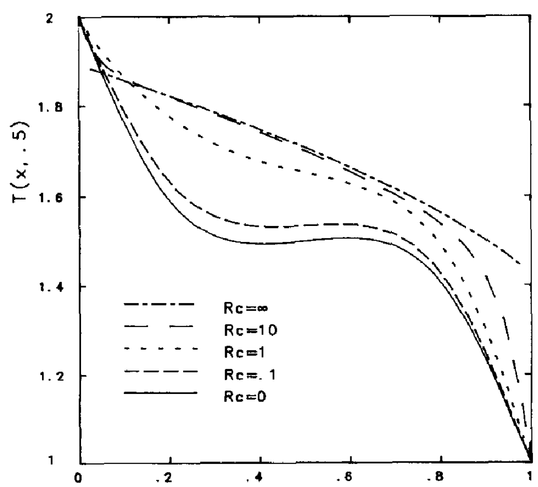


FIG. 5. Effect of radiation on centerline temperature distribution, when $Ra = 10^4$.

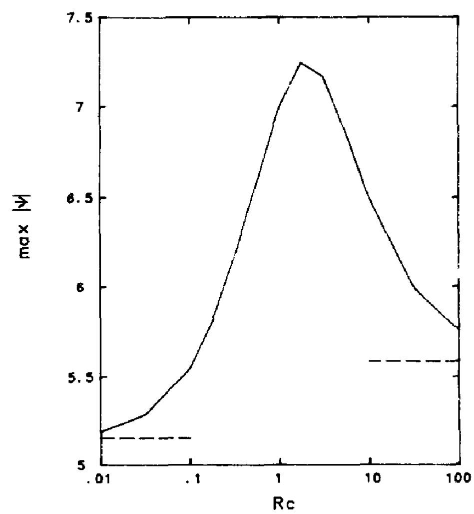


FIG. 6. Influence of radiation on maximum $|\psi|$ when $Ra = 10^4$. Dashed lines are for pure convection (left) and pure radiation (right) limits.

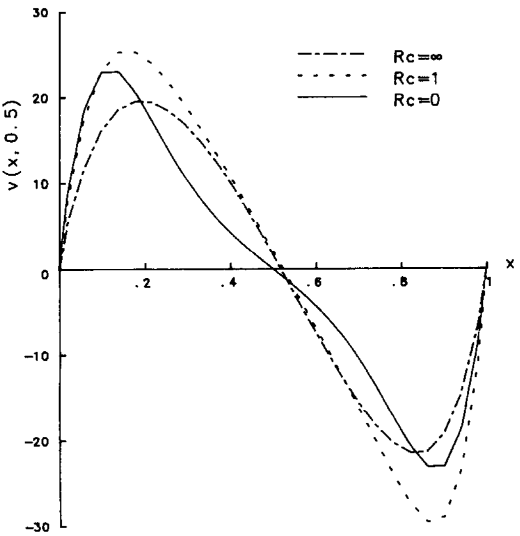


FIG. 7. Influence of radiation on vertical component velocity when $Ra = 10^4$.

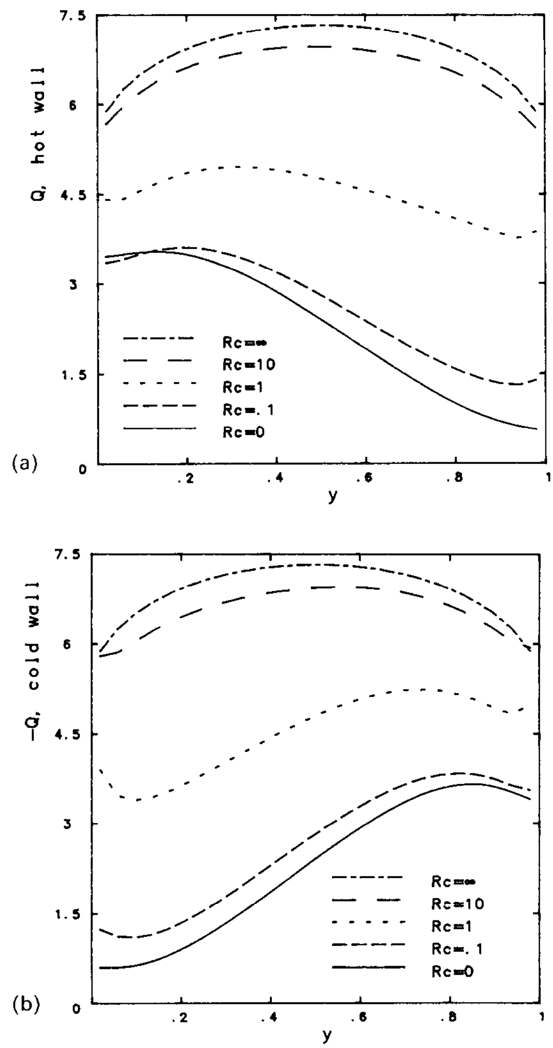


FIG. 8. Influence of radiation on heat flux distributions, when $Ra = 10^4$: (a) hot wall; (b) cold wall.

It is also noted that the fluxes are not constant in the pure radiation situation. This indicates the necessity to apply two-dimensional radiation models for the present problem.

Figure 9 shows the percentage radiation flux to the total heat transfer, when $Ra = 10^4$. The influence of radiation on heat transfer on the hot wall is more significant than on the cold wall, due to the fact that the radiation emissive power is proportional to T^4. At $Rc = 0.4$, the total radiative and convective fluxes are about the same. Figures 10(a) and (b) show the radiation fluxes on the upper and lower insulating walls, respectively.

It is interesting to find that when radiation is present, the isotherm lines are no longer orthogonal to the insulating walls as in the pure convection cases (Figs. 9(a) and (b)), indicating that both the radiative and conductive fluxes are nonzero on the insulating walls, while their total contribution is null. This is reasonable considering equation (3c). If a one-dimensional radiation model is used when both the radiation and convection are important, an unreal zero temperature gradient on the insulating walls is obtained. This shows the necessity of using a two-dimensional radiation model for the present problem, despite the fact that only a one-dimensional model is needed in the pure radiation case.

It is interesting to find that for the range of parameters examined in this work, the simple superposition for total heat flux using the flux for pure convection and pure radiation

$$Q'_{\text{comb}}(Rc) = \frac{q_c(Rc=0)+Rc\,q_r(Rc=\infty)}{1+Rc}$$

gives an excellent approximation (Fig. 11). Thus

$$q_c(Rc=0)+Rc\,q_r(Rc=\infty) \approx q_c(Rc)+Rc\,q_r(Rc).$$

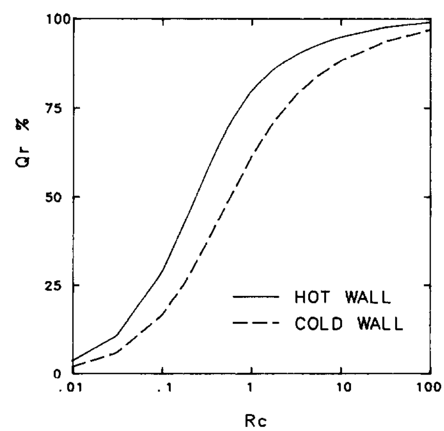


FIG. 9. Influence of radiation–conduction parameter on percentage radiation fluxes on hot and cold walls, when $Ra = 10^4$.

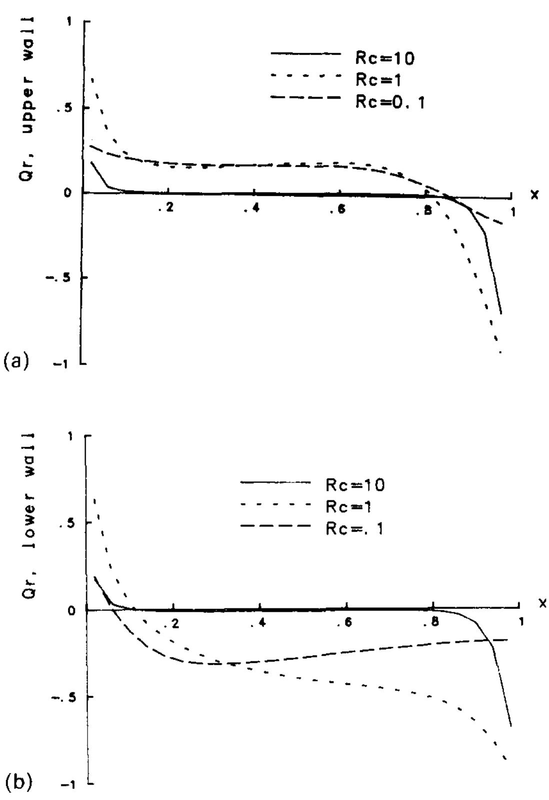


FIG. 10. Influence of radiation–conduction parameter on radiation flux distributions on insulating walls, when $Ra = 10^4$: (a) upper wall; (b) lower wall.

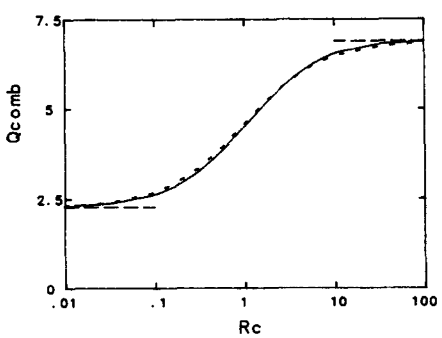


FIG. 11. Comparison of total heat flux by using the simple superposition formula (short dashed line) with the actual results (solid line). The long dashed lines are pure convection and radiation limits. $Ra = 10^4$.

Influence of optical properties on heat transfer

The influences of optical thickness, wall emissivity, and scattering albedo on heat transfer and fluid flow in the cavity were also studied for $Ra = 10^4$ and $Rc = 1$. The major results are listed in Table 1.

The influence of scattering on heat transfer is found to be very slight. In fact, when the scattering albedo varies from 0 to 0.9, the change in the scaled hot wall flux is only 0.0179, or 0.4%, while the change in the maximum stream function is 0.82, or 11.7%.

On the other hand, the influence of wall reflection on heat transfer is great: as the emissivity of the walls drops from 1 to 0.1, radiation fluxes on both the hot wall and the cold wall are greatly decreased to about 11 and 8%, respectively; the convective fluxes remain of the same order, resulting in a large change in the total flux (from 4.4987 to 1.9735). However, the maximum stream function is reduced by only 18%.

The effect of increasing optical thickness (from 0.1 to 10) is more significant. As we can see from the table, since the radiation fluxes are reduced drastically (as expected from radiation transfer theory), while the convection fluxes are almost unchanged, the total combined dimensionless heat flux on the wall is reduced from 6.0005 to 2.0549.

Numerical accuracy and efficiency

The major numerical errors of the present solutions are caused by finite difference truncation errors for the Navier–Stokes equations, and interpolation errors for the radiation transport equations. In pure convection cases, the present results give average heat fluxes of 1.118 for $Ra = 10^3$, 2.264 for $Ra = 10^4$ and 4.693 for $Ra = 10^5$, compared with benchmark solutions 1.118, 2.243 and 4.519 [3]. For pure radiation, the error by the present node number (25 × 25) compared with 45 × 45 mesh results is less than 0.2%. We conclude that the error in this computation is mainly caused by the finite difference scheme, and is expected to be less than 2% when $Ra \leqslant 10^4$ and less than 5% when $Ra = 10^5$. Better results may be obtained if a higher order finite difference scheme or a more dense grid is used, especially near the boundaries.

The computing time for each case depends on the choices of initial guesses and the relaxation parameters. However, even with simple choices, the present method does not require a long time to achieve convergence. Generally, the time to complete the solution to a combined heat transfer problem requires 1–2 times the total time to compute a pure convection problem (with the same Ra) and a pure radiation problem separately. For example, when $Ra = 10^4$, $Rc = 1$, $\varepsilon = 1$, $\tau = 1$ and $\omega = 0$, if the initial guess was the pure convection data for $Ra = 10^3$, the relaxation parameters were $\alpha_T = 0.7$, $\alpha_\omega = 0.4$ and $\alpha_\psi = 0.7$, and the convection–radiation iterating ratio L (refer to ref. [12]) was 40, then the total time for pure convection is 214 s, for pure radiation, 234 s, and for combined convection and radiation, 464 s (which is about the same as 214+234 = 448 s) on the CDC 170/750 computer.

CONCLUSIONS

In this paper, combined radiation and natural convection in a square cavity was studied by a numerical method that incorporates two-dimensional radiative transfer. The following conclusions are obtained:

(1) The presence of radiation will increase the bulk

Table 1. Influence of scattering, reflection and optical thickness on heat transfer and maximum stream function. $Ra = 10^4$ and $Rc = 1$

			Hot			Cold		Upper		Lower		
ε	ω	τ	Q_r	Q_c	Q	Q_r	Q_c	Q_r	Q_c	Q_r	Q_c	max $\|\psi\|$
1	0	1	3.6039	0.8948	4.4987	−2.7646	−1.7380	0.0865	−0.0865	−0.3434	0.3433	6.9885
1	0.5	1	3.6484	0.8362	4.4846	−2.8689	−1.6285	0.1125	−0.1125	−0.4419	0.4419	6.7032
1	0.9	1	3.6457	0.8350	4.4808	−3.0049	−1.4889	0.1314	−0.1313	−0.5982	0.5982	6.1685
0.5	0	1	1.8793	1.2208	3.1001	−1.2604	−1.8438	0.0567	−0.0564	−0.1942	0.1944	6.3055
0.1	0	1	0.4117	1.5618	1.9735	−0.2176	−1.7552	0.0246	−0.0253	−0.0386	0.0387	5.7060
1	0	0.1	5.1027	0.8978	6.0005	−4.4190	−1.5853	0.1244	−0.1245	−0.6284	0.6284	6.0901
1	0	10	1.1596	0.8953	2.0549	−0.4917	−1.5536	0.0035	−0.0034	−0.0329	0.0330	7.8153

temperatures of the fluid, and may have a significant influence on the fluid flow and temperature distributions. The influence of radiation on the total circulating mass varies at different Rayleigh numbers.

(2) All radiation parameters studied in this paper, except for the scattering albedo, have a significant influence on the heat transfer.

(3) To accurately simulate the temperature variations near the insulating walls, a two-dimensional radiation model is necessary.

(4) The numerical method used here is efficient. The results are quite accurate compared with benchmark solutions in limiting cases. To obtain more accurate results, a higher order finite difference scheme or more grid points should be applied.

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RAYONNEMENT ET CONVECTION NATURELLE COUPLES DANS UN MILIEU BIDIMENSIONNEL CARRE

Résumé—On étudie numériquement le rayonnement et la convection naturelle couplé dans un milieu bidimensionnel carré qui émet, absorbe et diffuse isotropiquement. La formulation intégrale exacte pour les équations du transfert radiatif et des bilans de quantité de mouvement et d'énergie sont discrétisées par une méthode produit-intégrale et aussi une méthode aux différences finies. Les équations algébriques résultantes sont résolues par une technique SOR non linéaire, avec le nombre de Rayleigh variant de 10^3, 10^4 à 10^5 et le paramètre rayonnement–conduction allant de zéro à l'infini. On discute les influences de ces paramètres et d'autres sur les distributions de vitesse et de température ainsi que le transfert thermique.

STRAHLUNG UND NATÜRLICHE KONVEKTION IN EINEM ZWEIDIMENSIONALEN STRAHLUNGSFÄHIGEN QUADRATISCHEN MEDIUM

Zusammenfassung—Die kombinierten Vorgänge von Strahlung und natürlicher Konvektion in einem zweidimensionalen, emittierenden, absorbierenden und isotrop streuenden, quadratischen Medium werden numerisch untersucht. Die exakten integralen Gleichungen für den Strahlungstransport und für das Impuls- und Energiegleichgewicht werden mit einem Produkt-Integral-Verfahren bzw. mit einem Finite-Differenzen-Verfahren diskretisiert. Es ergeben sich algebraische Gleichungen, die mit einem nichtlinearen SOR-Verfahren gelöst werden, und zwar für die Rayleigh-Zahlen 10^3, 10^4 und 10^5 sowie den gesamten Bereich des Strahlungs-Leitungs-Parameters von 0 bis ∞. Die Einflüsse des Strahlungs-Leitungs-Parameters, der Rayleigh-Zahl und anderer Parameter auf die Verteilung von Geschwindigkeit und Temperatur sowie auf den Wärmeübergang werden diskutiert.

ВЗАИМОСВЯЗАННЫЕ ИЗЛУЧЕНИЕ И ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ДВУМЕРНОЙ АКТИВНОЙ СРЕДЕ КВАДРАТНОЙ ФОРМЫ

Аннотация—Численно решена двумерная задача о совместном действии излучения и естественной конвекции в излучающей, поглощающей и изотропно рассеивающей среде, находящейся в полости квадратного сечения. Дискретные аналоги точного интегрального описания радиационного переноса, а также уравнений баланса импульса и энергии получены с помощью методов интегральных произведений и конечных разностей. Для решения результирующих алгебраических уравнений привлечен нелинейный метод SOR, причем число Рэлея изменялось в диапазоне от 10^3, 10^4 до 10^5, а радиационно-кондуктивный параметр изменялся в диапазоне от 0 до ∞. Обсуждается влияние радиационно-кондуктивного параметра, числа Рэлея и других параметров на режим течения, температурное поле и теплоперенос.